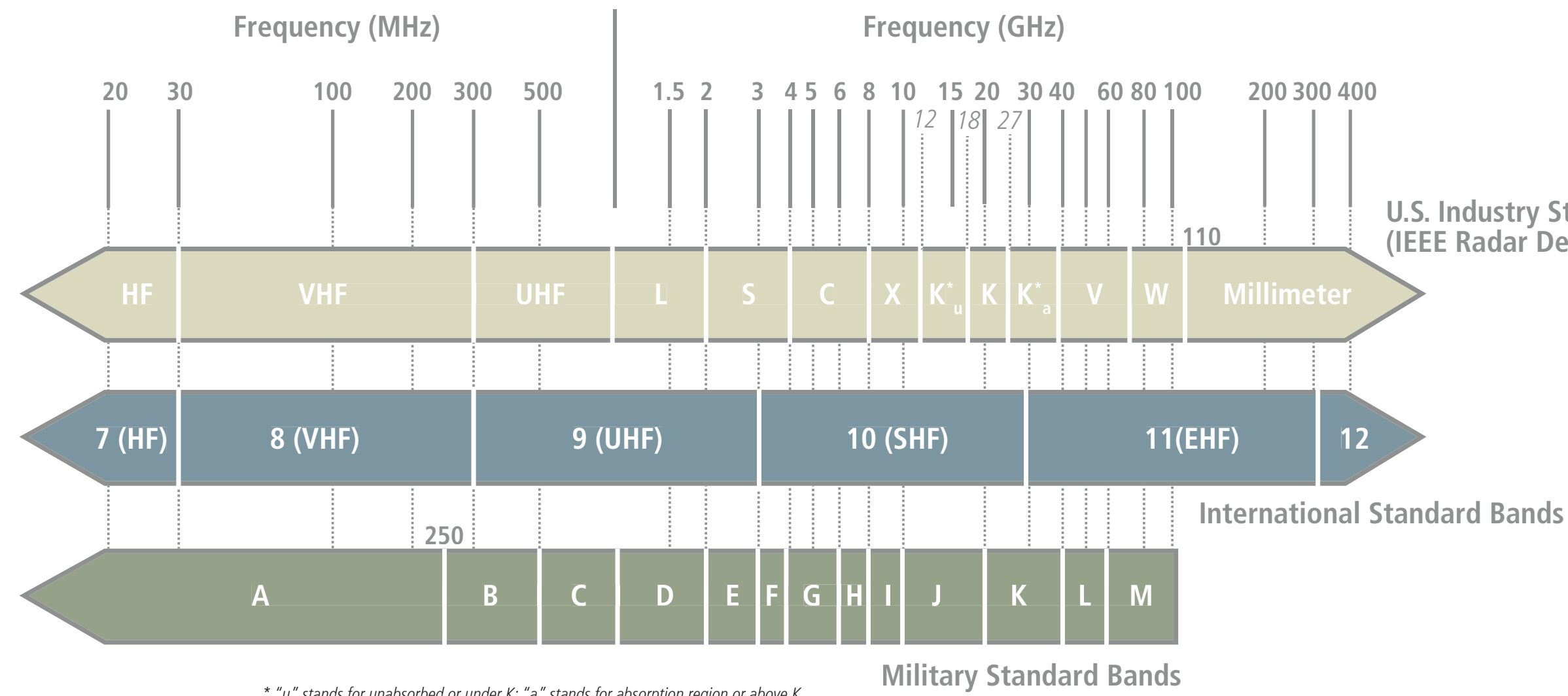
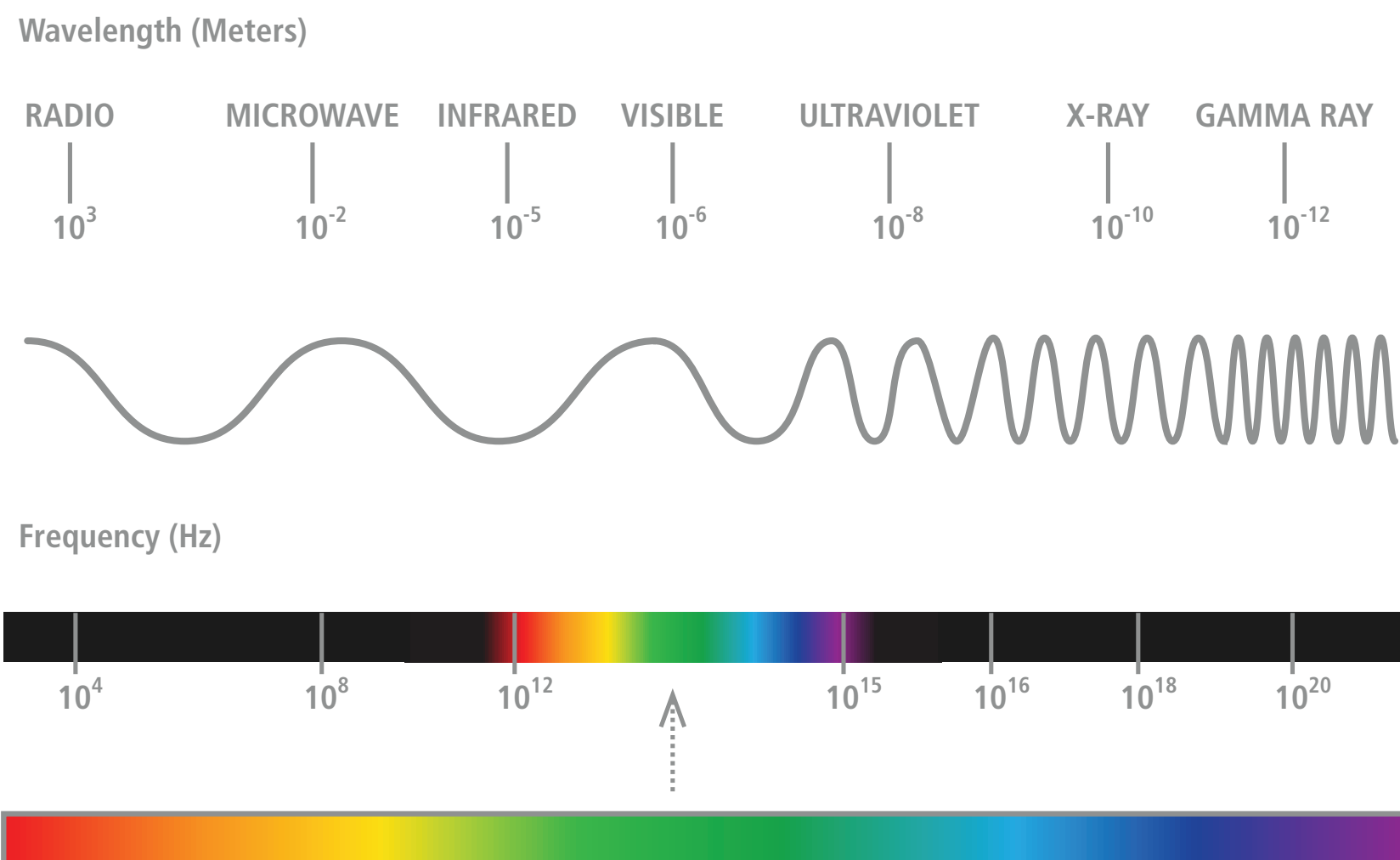


ELECTRONIC WARFARE

QUICK REFERENCE GUIDE

THE ELECTROMAGNETIC SPECTRUM



Band Designation	Frequency Range
HF	3-30 MHz
VHF	30-300 MHz
UHF	300-1,000 MHz
L	1-2 GHz
S	2-4 GHz
C	4-8 GHz
X	8-12 GHz
K _u	12-18 GHz
K	18-27 GHz
K _a	27-40 GHz
V	40-75 GHz
W	75-110 GHz

RF Propagation

FRIS TRANSMISSION EQUATION

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2$$

P_r: Received Power
P_t: Transmit Power
G_t: Transmit Gain
G_r: Receive Gain
R: Range

RF Propagation

RADAR HORIZON

$$D_h = \sqrt{2HR_e}$$

H: Horizon
R_e: Earth Radius - 6,371 km

RF Propagation

TARGET VISIBILITY

$$\text{Target Height} = \frac{(\text{Target Range} - \sqrt{2HR_e})^2}{2Re}$$

H: Horizon
R_e: Earth Radius - 6,371 km

Detection & Estimation Probability

MAX LIKELIHOOD ESTIMATION

Joint Density Function

$$f(x_1, x_2, \dots, x_n | \theta) = f(x_1 | \theta) \times f(x_2 | \theta) \times \dots \times f(x_n | \theta)$$

Likelihood

$$L(\theta; x_1, \dots, x_n) = f(x_1, x_2, \dots, x_n | \theta) = \prod_{i=1}^n f(x_i | \theta)$$

Log-Likelihood

$$\ln L(\theta; x_1, \dots, x_n) = \sum_{i=1}^n \ln f(x_i | \theta)$$

Average Log-Likelihood

$$\ell = \frac{1}{n} \ln L$$

$$\hat{\ell} = \frac{1}{n} \sum_{i=1}^n \ln f(x_i | \theta)$$

x_i: Observations
n: Number of Samples
f: Is one or joint probability distribution(s)
θ: Distribution parameters can be vectors

Electronic Warfare

NOISE JAMMING

$$S = \frac{EIRP_{radar} G_{radar} G_r \lambda^2}{(4\pi R)^3 R^4} \sigma$$

$$J/N \sim \left(\frac{R_{max}}{R_{max,jammed}}\right)^4$$

Assume: *J* >> *N*
BW_{jam} = *BW_{radar}*

$$R_{max,jammed}^4 = \frac{P_t G_t G_r \lambda^2}{(4\pi)^3 (k T_s B_N N_f + J) * SNR * L_r * L_t}$$

Mainlobe

$$\frac{J}{S} = \frac{EIRP_{jam}}{EIRP_{radar}} \left(\frac{\lambda}{4\pi R}\right)^2 \frac{G_{radar}}{L_r}$$

If *BW_{jam}* > *BW_{radar}*

$$\frac{J}{S} = \left(\frac{EIRP_{jam}}{EIRP_{radar}}\right) \left(\frac{4\pi R^2}{\sigma}\right) \left(\frac{BW_{radar}}{BW_{jam}}\right)$$

Reduction in Normalized R_{max}

Reduction in Radar Detection Range due to JNR

Side-lobe

Reduction in Radar Detection Range due to JNR

J_{self}: Self Protect Jammer Power
J_S: Jam to Signal Ratio at Radar Receiver
S: Radar Received Signal Power
P_{jam}: Jammer Transmit Power
G_{jam}: Jammer Transmit Gain
R_J: Range between Jammer and Radar
R_T: Range between Radar Target and Radar
k: Jammer Transmit Wavelength
G_{radar}: Radar Receiver Gain

L_{radar}: Radar Receiver Losses
P_{radar}: Radar Transmit Power
S_{radar}: Radar Receiver Signal Power
G_{radar}: Radar Transmitter Gain
σ: Radar Target Radar Cross Section
BW_{radar}: Radar Transmit Bandwidth
BW_{jam}: Jammer Transmit Bandwidth
J: Jammer Power
R_{max,jammed}: Jammed Radar Range (Burns through Range)

R_{max}: Max Radar Range
J/N: Jammer to Noise Ratio
N: Total Noise
k: Boltzmann's constant
T_s: Receiver Temperature
B_N: Receiver Noise Bandwidth
SNR: Radar Signal to Noise Ratio
N_f: Receiver Noise Figure (>1)

Radar Processing

LINEAR FM WAVEFORM

$$s(\tau) = e^{i2\pi(f_c + \frac{1}{2} b\tau^2)}, -\frac{\tau_p}{2} \leq \tau \leq \frac{\tau_p}{2}$$

$$B_p = b\tau_p$$

b: Frequency Chirp Rate
τ_p: Pulse Length

determines resolution

determines signal energy

s_c: Transmitted Signal Waveform
f_c: Center Frequency
τ_p: Pulse Length
b: Chirp Rate
B_p: Pulse Bandwidth
τ_p: Range Frequency

Radar Processing

RADAR AMBIGUITY FUNCTION

$$x(\tau, t) = \int_{-\infty}^{\infty} s(t) s^*(t - \tau) e^{i2\pi f t} dt$$

S_c: Complex Baseband Pulse
τ: Time Delay
f: Doppler Shift

Radar Processing

NOISE POWER

$$\text{Noise Power in Receiver} = k T_s B_N N_f$$

kT_s = -174 dBm
k: Boltzmann's constant = 1.38 × 10⁻²³ J/K
B_N: Noise Bandwidth
T_s: System Noise Temperature
T_s usually set to *T_{amb}* = 290K
N_f: Noise figure of receiver

RF Propagation

WAVELENGTH

$$\lambda = \frac{c}{f}$$

Band	f	Wavelength
VHF	100 MHz	3.00 m
S	3 GHz	0.10 m
C	6 GHz	0.05 m
X	10 GHz	0.03 m

c: Speed of Light
f: Frequency

RF Propagation

DOPPLER SHIFT

$$f_d = -2v_r / \lambda$$

X-band	S-band	
Velocity	300 m/s	300 m/s
Wavelength	0.03 m	0.1 m
Doppler Shift	20 kHz	6 kHz

Detection & Estimation Probability

CRAMER RAO LOWER BOUND

$$CRB = \left(E \left[\left[\frac{\partial \ln p(x, \theta)}{\partial \theta} \right] \left[\frac{\partial \ln p(x, \theta)}{\partial \theta} \right]^T \right] \right)^{-1}$$

x: Observations
p: Probability distribution function (or joint)
θ: Distribution parameters can be vectors

Detection & Estimation Probability

BINOMIAL

$$f(k; n, p) = Pr(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

p: Success probability of each trial
k: Number of successes
n: Number of trials

Detection & Estimation Probability

RAYLEIGH

$$p(r) = \begin{cases} \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} & (r > 0) \\ 0 & (0 \leq r \leq \infty) \end{cases}$$

σ: Standard Deviation
A: Distance between the reference point and the center of the bivariate distribution

Detection & Estimation Probability

ERROR FUNCTIONS

$$\text{erfc}(z) = 1 - \text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-t^2} dt$$

$$\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$$

μ: Mean
σ: Standard Deviation
A: Distance between the reference point and the center of the bivariate distribution

Fourier Relationships

CONTINUOUS-TIME FOURIER TRANSFORMATION

Synthesis

$$x(t) = \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Analysis

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

x(t) ↔ *X(ω)*

Fourier Relationships

MODULATION PROPERTY

Modulation

$$s(t) p(t) \xleftrightarrow{FT} \frac{1}{2\pi} [S(\omega) P(\omega)]$$

Convolution

$$h(t) * x(t) \xleftrightarrow{FT} H(\omega) X(\omega)$$

Time Shifting

$$x(t - t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(\omega)$$

Differentiation

$$\frac{dx(t)}{dt} \xleftrightarrow{FT} j\omega X(\omega)$$

Integration

$$\int_{-\infty}^t x(\tau) d\tau \xleftrightarrow{FT} \frac{1}{j\omega} X(\omega) + \pi X(\omega) \delta(\omega)$$

Linearity

$$ax_1(t) + bx_2(t) \xleftrightarrow{FT} aX_1(\omega) + bX_2(\omega)$$

Duality Property

Radar Processing

MAX UNAMBIGUOUS RANGE

$$R_{max} = \frac{c}{2PRF}$$

PRF	Range	Doppler	PRF	Unambiguous Range
High	Ambiguous	Unambiguous	100 kHz	1.5 km
Medium	Ambiguous	Ambiguous	25 kHz	6 km
Low	Unambiguous	Ambiguous	10 kHz	15 km

c: Speed of Light
PRF: Pulse Repetition Frequency

Detection & Estimation Probability

RICIAN

$$p(r) = \begin{cases} \frac{r}{\sigma^2} e^{-\frac{r^2 + A^2}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right) & \text{for } (A \geq 0, r \geq 0) \\ 0 & \text{for } (r < 0) \end{cases}$$

μ: Mean
σ: Standard Deviation
A: Distance between the reference point and the center of the bivariate distribution
I₀: Bessel Function of the first kind with order zero

Detection & Estimation Probability

NORMAL

$$p(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Standard Normal Curve (*μ*_z = 0; *σ*_z = 1.0)

*f_z(z) = 1/√(2π) * e^(-z^2/2)*

±1-σ: P(-1 ≤ z ≤ 1) = 0.6827
±2-σ: P(-2 ≤ z ≤ 2) = 0.9545
±3-σ: P(-3 ≤ z ≤ 3) = 0.9973

μ: Mean
σ: Standard Deviation
A: Distance between the reference point and the center of the bivariate distribution

Fourier Relationships

FILTERING

Ideal Lowpass Filter

Differentiator

$$y(t) = \frac{dx(t)}{dt} \Rightarrow H(\omega) = j\omega$$

Convolution Property

$$h(t) * x(t) \xleftrightarrow{FT} H(\omega) X(\omega)$$

$$\frac{x(t)}{X(\omega)} \xleftrightarrow{FT} \frac{h(t) * x(t)}{H(\omega) X(\omega)} \Rightarrow \frac{\delta(t)}{1} \xleftrightarrow{FT} \frac{h(t)}{H(\omega)}$$

H(ω): Frequency Response
***: Convolution operation

Radar Processing

RADAR CROSS SECTION

$$\sigma = \frac{\text{Reflected Power to Receiver} / \text{Solid Angle}}{\text{Incident Power Density} / 4\pi} = \lim_{r \rightarrow \infty} 4\pi r^2 \left(\frac{|E_s|^2}{|E_i|^2} \right)$$

P_t, or *S*
S < *c*, range
Radar Cross Section (RCS, σ)
Scattering

Radar Processing

SIGNAL TO NOISE RATIO

$$SNR = \frac{P_R}{N_o} = \frac{P_t G_t G_r \sigma \lambda^2 G_p L}{(4\pi)^3 R^4 k T_s B_N N_f}$$

P_r: Received Power
P_t: Transmit Power
G_t: Transmit Gain
G_r: Receive Gain
R: Range
N_o: Noise Power
L: Losses

k: Boltzmann's constant = 1.38 × 10⁻²³ J/K
B_N: Noise Bandwidth
T_s: System Noise Temperature
T_s usually set to *T_{amb}* = 290K
N_f: Noise figure of receiver

Radar Processing

TYPICAL VALUES OF RCS

Human
Small Car
Fighter Aircraft
Bomber Aircraft
Ships